



DCMT101

Reg. No.

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I Semester B.Sc. Degree Examination, May/June - 2022

MATHEMATICS

Algebra - I and Calculus - I

(NEP - CORE Scheme 2021-22 and Onwards)

Paper :I MATDSCT 1.1

Time : 2½ Hours

Maximum Marks : 60

*Instructions to Candidates:*

Answer all questions.

I. Answer any Six questions.

(6×2=12)

1. Find the value of k in order that the matrix

$$A = \begin{bmatrix} 6 & k & -1 \\ 2 & 3 & 1 \\ 3 & 4 & 2 \end{bmatrix} \text{ is of rank 2.}$$

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Bangalore-560 0112. Find the eigen values of the matrix  $\begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix}$ .3. Find the  $n^{\text{th}}$  derivative of  $e^{2x} \cos 3x$ .4. If  $u = x^2 yz$  prove that  $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$ .5. Evaluate  $\lim_{x \rightarrow 0} x \sin\left(\frac{1}{x}\right)$ .

6. State Lagrange's mean value theorem.

7. Discuss the continuity of  $f(x) = \begin{cases} 3x+1 & x > 1 \\ 2x-1 & x \leq 1 \end{cases}$  at  $x = 1$ .8. Evaluate  $\lim_{x \rightarrow 0} \frac{x - \sin x}{x^3}$ .

[P.T.O.]



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(3×4=12)**II. Answer any Three questions.**

9. Find the rank of the matrix  $A = \begin{bmatrix} 1 & 1 & 1 & 2 \\ 2 & 1 & -3 & -6 \\ 3 & -3 & 1 & 2 \end{bmatrix}$  by reducing it to normal form.

10. Find  $\lambda$  and  $\mu$  such that the system of equations

$$x + 3y + 4z = 5$$

$$x + 2y + z = 3 \quad \text{has}$$

$$x + 3y + \lambda z = \mu$$

- i. no solution
- ii. unique solution.
- iii. many solutions.

11. Find the eigen values and the corresponding eigen vectors of the matrix.

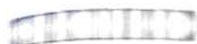
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$$

12. State and prove Cayley - Hamilton theorem.

13. By using Cayley - Hamilton theorem. Find the adjoint of the matrix  $\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix}$ .

**III. Answer any Three questions.****(3×4=12)**

14. Discuss the continuity of  $f(x) = \begin{cases} x^2 - 1 & \text{for } x < 1 \\ 1 - \frac{1}{x} & \text{for } x > 1 \\ 0 & \text{for } x = 1 \end{cases}$  at  $x = 1$ .



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15. Examine the differentiability of  $f(x) = \begin{cases} x^2, & x < 3 \\ 6x - 9, & x \geq 3 \end{cases}$  at  $x = 3$ .
16. Find the  $n^{\text{th}}$  derivative of  $\frac{4x}{(x+1)^2(x-1)^2}$ .
17. Prove that a function which is continuous in a closed interval attains its bounds in the interval.
18. If  $y = \sin(n \sin^{-1} x)$ , prove that  $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2 - m^2)y_n = 0$ .

IV. Answer any Three questions.

(3×4=12)

19. State and prove Rolle's theorem.
20. State and prove Cauchy's Mean value theorem.
21. Expand  $\log(1+\sin x)$  upto the term containing  $x^4$  using Maclaurin's series.
22. Evaluate  $\lim_{x \rightarrow 0} \frac{\tan x - \sin x}{\sin^3 x}$ .
23. Evaluate  $\lim_{x \rightarrow 0} (1 + \sin x)^{\cot x}$ .

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V. Answer any Three questions.

(3×4=12)

24. If  $u = f(r)$  where  $r = \sqrt{x^2 + y^2 + z^2}$  show that  $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = f''(r) + \frac{2}{r} f'(r)$ .
25. State and prove Euler's theorem on Homogeneous function.
26. If  $u = x^2, v = y^2$  find  $J = \frac{\partial(u,v)}{\partial(x,y)}$  and  $J' = \frac{\partial(x,y)}{\partial(u,v)}$  also verify that  $JJ' = 1$ .
27. Expand  $e^x \cos y$  in a Taylor's series about the point at  $(1, \pi/4)$  upto second degree terms.
28. Find the extreme value of the function  $f(x, y) = x^3 + y^3 - 3x - 12y + 2$ .