



DCMT201

Reg. No.

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II Semester B.Sc. Degree Examination, Oct.- 2022

MATHEMATICS

Algebra - II and Calculus - II

(NEP-CORE)

Paper: II

Time : 2½ Hours

Maximum Marks : 60

Instructions to Candidates:

Answer all the questions

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I. Answer any Six questions.

(6×2=12)

1. In the group of integers \mathbb{Z} an operation $*$ is defined by $a*b = a+b-3 \quad \forall a, b \in \mathbb{Z}$ find $(5*2)^{-1}$
2. Prove that every group of order 3 is abelian.
3. Prove that every subgroup of an abelian group is normal.
4. Show that $f : (G, +) \rightarrow (G', \cdot)$ defined by $f(x) = e^x, \forall x \in G$ is a homomorphism.
5. Find the radius of curvature at any point (p, r) on the curve $r^3 = a^2 p$
6. Find $\frac{ds}{dx}$ for the curve $y^2 = 4ax$.
7. Evaluate $\int_0^{\pi/2} \cos^5 x \, dx$
8. Find the asymptotes parallel to x -axis of the curve $y^3 - x^2 y + 2y^2 + 4y + 1 = 0$

II. Answer any THREE questions.

(3×4=12)

9. Prove that set Q_1 of rational numbers other than 1 the binary operation $a*b = a+b-ab$ is an abelian group.
10. Prove that a non-empty subset H of a group G is a subgroup of G if and only if $ab^{-1} \in H, \forall a, b \in H$.

[P.T.O.]



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11. G is a group and $a \in G$ is an element of order n , prove that for any positive integer m , $a^m = e$ if and only if n is a divisor of m .
12. If H is a subgroup of a group G prove that there is a one to one correspondence between the set of all right cosets and the set of all left cosets of H in G .
13. State and prove Lagrange's theorem in groups.

III. Answer any THREE questions.

(3×4=12)

14. Prove that a subgroup H of a group G is normal if and only if $ghg^{-1} \in H, \forall g \in G, h \in H$.
15. Prove that the product of two normal subgroups of a group is a subgroup of a group.
16. Let $(\mathbb{Z}, +)$ be the additive group of all integers and $H = \{1, -1\}$ be the multiplicative group. Define $f(z) = \begin{cases} 1 & \text{if } z \text{ is even} \\ -1 & \text{if } z \text{ is odd} \end{cases}$

Show that f is a homomorphism.

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17. If $f: G \rightarrow G'$ is a homomorphism then prove that
 - i) $f(e) = e'$, where e and e' are identity elements in G and G' respectively and
 - ii) $f(a^{-1}) = (f(a))^{-1}, \forall a \in G$
18. State and prove fundamental theorem of homomorphism of group.

IV. Answer any THREE questions.

(3×4=12)

19. Show that the curves $r^n = a^n \cos n\theta$ and $r^n = b^n \sin n\theta$ intersect orthogonally.
20. With usual notations prove that $p = r \sin \phi$ and $\frac{1}{p^2} = \frac{1}{r^2} + \frac{1}{r^4} \left(\frac{dr}{d\theta} \right)^2$
21. Find the radius of curvature of the cycloid $x = a(\theta + \sin \theta), y = a(1 + \cos \theta)$
22. Find the circle of curvature for the curve $xy = c^2$ at (c, c) .
23. Find all the asymptotes to the curve $x^3 + 3x^2y - 4y^3 - x + y + 3 = 0$

V. Answer any THREE questions.

(3×4=12)

24. Find the reduction formula for $\int \sin^n x \, dx, n > 0$.

25. Evaluate: $\int_0^{\pi} x \sin^2 x \cos^2 x \, dx$

26. Find the area of the cardioid $r = a(1 + \cos \theta)$

27. Find the length of the arc of the parabola $y^2 = ax$ cut off by the latus rectum.

28. Find the volume of the solid generated by revolving $r^2 = a^2 \cos 2\theta$ about the line $\theta = \frac{\pi}{4}$

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